Reflection coefficient and reflection angle ... what do they mean?

By Harold Kinley, C.E.T.

In radio electronics, there are many ways to express the same thing. *Reflection coefficient* is a way of expressing the degree of impedance match or mismatch between a load and a transmission line or between a transmission line and a generator. The angle associated with the reflection coefficient varies along a transmission line. The *angle* of the reflection coefficient, as well as the *amplitude* of the reflection coefficient, determines the *magnitude* of the standing wave at a particular point along a transmission line.

Reflection coefficient is normally represented by the Greek letter Γ (gamma).*

*Sometimes, reflection coefficient is represented by the Greek letter r (rho).

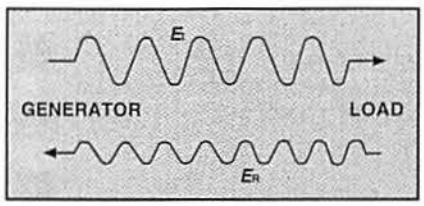

Figure 1. When a load is not properly matched to the transmission line, a reflected wave travels down the line in a direction opposite to the forward wave. The phase relationship of the two waves varies along the length of the line, repeating every half-wavelength. The resultant voltage depends on the magnitude and phase relationship between the two waves. When the two waves are in phase (0°), a voltage maximum occurs on the standing wave. Conversely, when the two waves are 180° out of phase, a voltage minimum occurs on the standing wave.

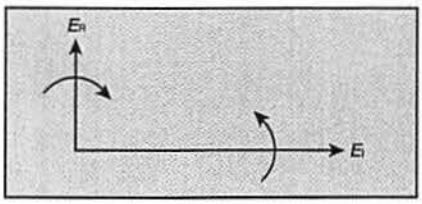

Figure 2. Two phasors represent the incident and reflected waves. The two phasors rotate in opposite directions at the same speed of rotation. This rotation causes variations in the amplitude of the *resultant* standing wave along the line.

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The VSWR (voltage standing wave ratio) can be determined from Γ by using Equation 1 in the box at the right.

When a transmission line terminates in an impedance (Z_L) that is not equal to the transmission line characteristic impedance (Z_0), a reflected wave appears on the line. The voltage of the reflected wave (E_R) will travel from the load toward the generator. This movement is opposite in direction to the forward or *incident* wave (E_I). (See Figure 1, below left.)

The phase relationship between E_I and E_R varies at different points along the line. This variation is represented by phasors in Figure 2, bottom left. As shown, E_I rotates counterclockwise, and E_R rotates clockwise. Each phasor rotates at the same speed but in an opposite direction. This difference causes the phasors to add and subtract according to the phase relationship between them at any given point.

Complex impedance mismatch

Consider a complex load of $20 - j80$ ohms connected to a 50Ω transmission line. How do we calculate the reflection coefficient and the angle of reflection? Use Equation 2. The calculations are shown in the box below. The load impedance (Z_L) must be entered in complex form. The magnitude of the numerator and denominator is then found by taking the square root of the sum of the squares of the *absolute values* (disregarding signs) of the *real* and *imaginary* components. The angles are determined from the tangents. For example, for the numerator the tangent is $-40/-15 = 2.666$.

$$VSWR = \frac{1+\Gamma}{1-\Gamma} \quad [1]$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad [2]$$

$$\phi = \theta - (720\lambda) \quad [3]$$

$$E = \sqrt{(\Gamma\cos\phi + 1)^2 + (\Gamma\sin\phi)^2} \quad [4]$$

$$Z_N = \frac{1+\Gamma}{1-\Gamma} \quad [5]$$

$$R_L = 20\log\Gamma \quad [6]$$

$$\%REF.POWER = 100\Gamma^2 \quad [7]$$

$\text{Tan}^{-1}(2.666) = 69.44°$. Figure 3 on page 72 shows that the result lies in the third quadrant, and so we must add 180° to 69.44° to get 249.44°. The denominator is calculated similarly to get a magnitude of 53.15 at an angle of −48.81°. Finally, the reflection coefficient (Γ) is calculated to be 0.8 at an angle of 298.25°, or simply −61.7°. This is the reflection coefficient (Γ) at the load.

To find the relative voltage level of the standing wave at the load, use Equation 4. This calculation gives us a factor of 1.55 at the load. This factor means that the combined voltage of the forward wave and the reflected wave at the load

(continued on page 72)

$$\frac{Z_L - Z_0}{Z_L + Z_0} = \frac{20 - j80 - 50}{20 - j80 + 50} = \frac{-30 - j80}{70 - j80} = \frac{2(-15 - j40)}{2(35 - j40)} = \frac{-15 - j40}{35 - j40}$$

$$\tan^{-1}\left(\frac{-40}{-15}\right) = 69.4° \quad (69.4 + 180 = 249.4°)$$

$$\tan^{-1}\left(\frac{-40}{-35}\right) = -48.8°$$

$$Z_L - Z_0 = \frac{40}{\sin 69.4} = 42.7\angle 249.44°$$

$$Z_L + Z_0 = \frac{40}{\sin 48.8°} = 53.2\angle -48.81°$$

$$\frac{42.7\angle 249.44°}{53.2\angle -48.81°} = 0.8\angle 298.3° = 0.8\angle -61.7°$$

(continued from page 8)

is 1.55 times the voltage of the forward wave. This function of combining voltages is how standing waves produce higher voltage stresses on transmission lines. The higher the standing wave ratio, the higher the reflection coefficient and the higher the peak stress voltages on the line. A VSWR of infinity would produce a reflection coefficient of 1, and because the voltage of the reflected wave would equal the voltage of the forward wave, the maximum combined voltage of the two in phase would be twice the voltage of the forward wave alone. The combination of the two at 180° out of phase would be zero voltage. Thus the VSWR would be 2:0, or infinity.

Because the perfect short circuit or perfect open circuit does not exist, the VSWR can never reach infinity. However, it can be very high, and as a result it can produce very high peak voltages at points where the voltages of the forward and reflected waves combine in phase.

To determine the angle of reflection at any point of the transmission line, use Equation 3. Then, to find the relative voltage of the standing wave at that point,

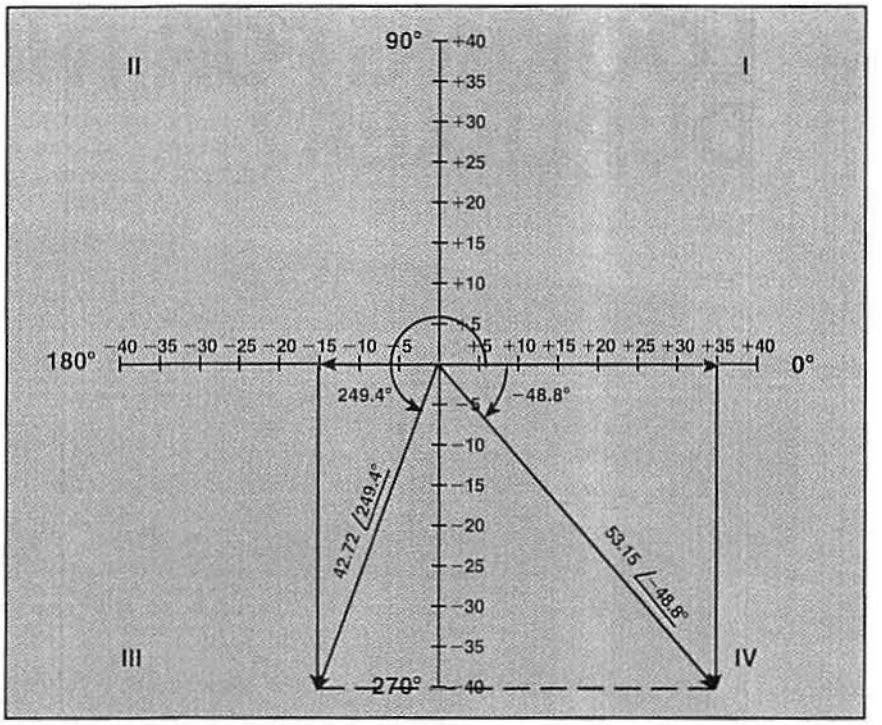

Figure 3. Impedance is converted to polar form from the rectangular form. The resultants in polar form are then used to calculate the reflection coefficient and the angle of reflection. See text and equations for the procedure.

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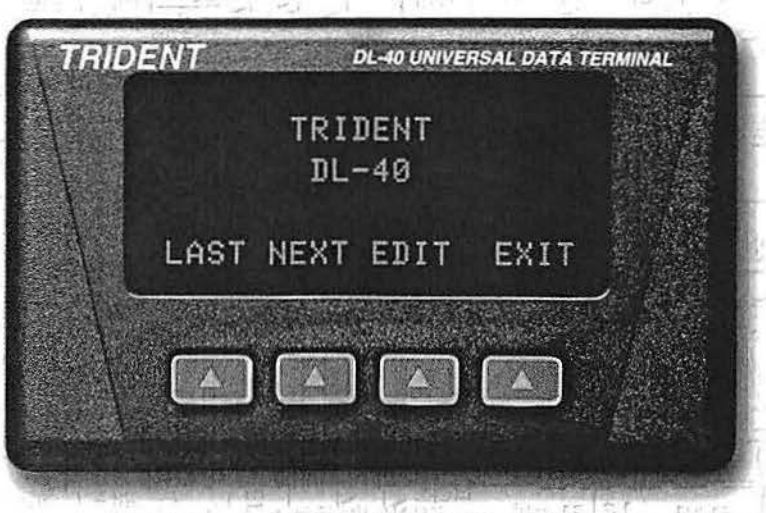

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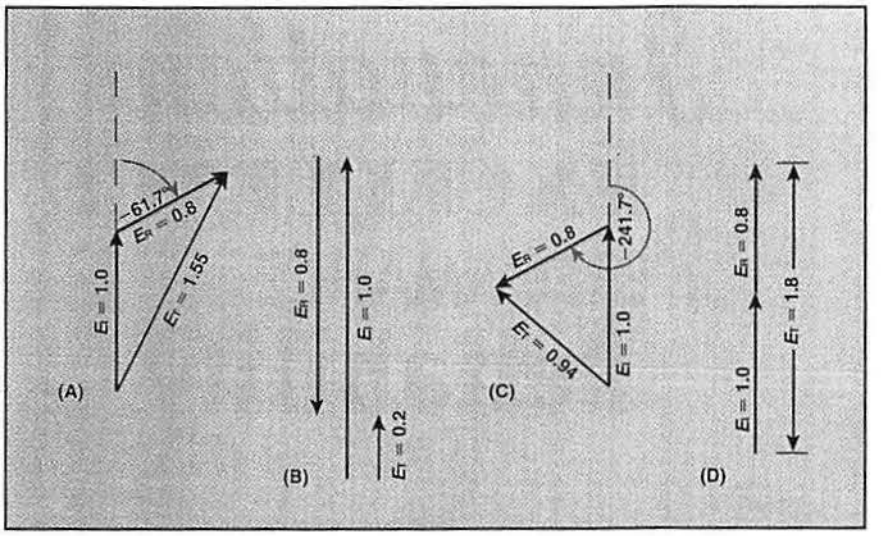


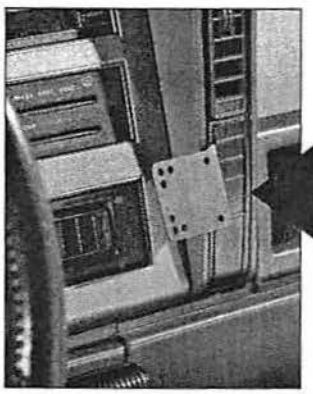
Figure 4. This phasor representation of the incident and reflected waves shows how the phase relationships and magnitude produce a varying level of standing wave.

substitute this angle (phi or ϕ) into Equation 4. Theta (θ) in Equation 3 is the reflection angle at the load.

Figure 4 above shows how the reflected and forward waves combine at various points. The phasors in examples A and C occur at 0.25λ apart on the line. Also, examples B and D are 0.25λ apart. Note that in example B the phasors are 180° out of phase and subtract directly. This produces the voltage minimum on the standing wave. In example D, the phasors are 0° apart (exactly in phase) and therefore add directly. This summing produces

Table 1. Complex load impedance, where (Z_L) = 20 − *j*80 ohms. Reflection coefficient (Γ) = 0.8 at an angle of −61.7° at load. VSWR = 9:1.

	Distance from load (λ)	Angle of reflection (°)	Magnitude of standing wave
	0	−61.7	1.55
	0.025	−79.9	1.39
	0.050	−97.7	1.19
	0.075	−115.7	0.97
	0.100	−133.7	0.73
	0.125	−151.7	0.48
	0.150	−169.7	0.26
(MIN)	0.1645	−180	0.20
	0.175	−187.7	0.23
	0.200	−205.7	0.45
	0.225	−223.7	0.70
	0.250	−241.7	0.94
	0.275	−259.7	1.16
	0.300	−277.7	1.36
	0.325	−295.7	1.53
	0.350	−313.7	1.66
	0.375	−331.7	1.75
	0.400	−349.7	1.79
(MAX)	0.4145	−360	1.80
	0.425	−367.7	1.796
	0.450	−385.7	1.76
	0.475	−403.7	1.67
	0.500	−421.7	1.55



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the voltage maximum on the standing wave.

Table 1 on page 74 shows the relative magnitude of the standing wave at various points on the transmission line. Note that the minimum and maximum values fall exactly 0.25λ apart. Figure 5 at the right shows a plot of the standing wave that represents the load of 20 − *j*80 ohms.

Calculating impedance (Z)

The impedance (Z) at any point on a line can be calculated from the Γ and the reflection angle. First, Γ must be converted to the complex form of $R \pm jX$. $R = \Gamma\cos(\phi)$ and $jX = \Gamma\sin(\phi)$. Use Equation 5 to calculate Z. Remember, Γ must be in complex form ($R \pm jX$).

For example, using Table 1, calculate the complex impedance at a point 0.375λ from the load. Here the angle of reflection (θ) is −331.7°. The calculations are shown in the box on page 78. Note that Z_N is normalized impedance such as you would see on a Smith chart. To get the impedance in a 50Ω system, multiply by 50, as shown on page 78.

Return loss

Return loss can be easily calculated

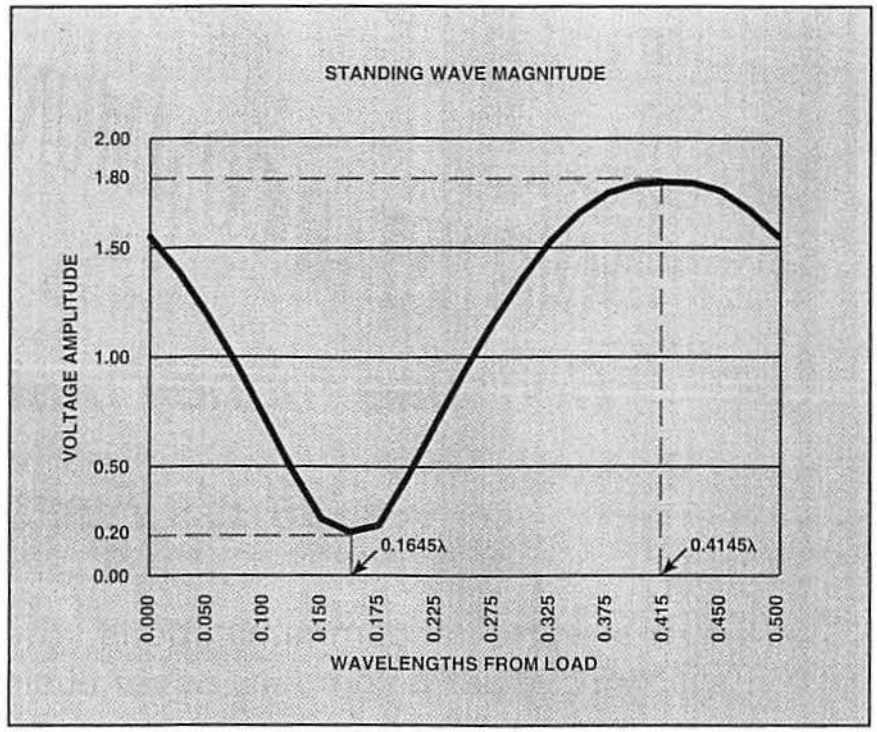

Figure 5. The results of Table 1 produce a graphic representation of the standing wave on the transmission line loaded by the 20 − j80 ohm load. Note that the voltage minimum and maximum occur at 0.25λ apart.

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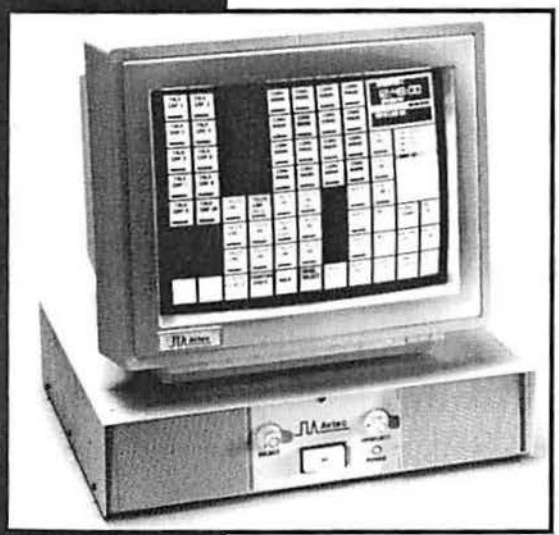

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from reflection coefficient. For example, the reflection coefficient of 0.8 as used in the above example would correspond to a return loss of 1.94dB. Use Equation 6. The percentage of reflected power can be determined from Equation 7. For example, a reflection coefficient of 0.8 is equivalent to 64% reflected power. If a directional wattmeter indicated a forward power of 100W, then the reflected power would indicate 64W.

Summing up

As you can see, there are many ways to represent the same characteristics on a transmission line. There are situations when it is easier or better to indicate mismatch in terms of Γ, VSWR, percentage reflected power or return loss. At other times, it is simply a matter of personal preference. No matter how it is represented, this should help you to understand how the various units of measure are interrelated.

As usual, a computer program is available that will compute the reflection coefficient and reflection angle at the load or at any other point on a transmission line. A second program deals with VSWR and various conversions. Both programs are available on a single diskette for $7.50 plus $2.50 for shipping and handling. The programs run under MS-DOS on an IBM-compatible PC. Specify 3.5" or 5.25" diskette. You may write the author at 204 Tanglewylde Drive, Spartanburg, SC, 29301-2949.

'Til next time ... *stay tuned!*

Reflection coefficient (Γ) = 0.8 @ −331.7°

$R = 0.8\cos(-331.7°) = 0.8(0.88) = 0.704$

$jX = 0.8\sin(-331.7°) = 0.8(0.47) = 0.376$

$\Gamma = 0.704 + j0.376$

$$Z_n = \frac{1 + (0.704 + j0.376)}{1 - (0.704 + j0.376)} = \frac{1.704 + j0.376}{0.296 - j0.376} = 1.59 + j3.28$$

$Z = 50(1.59 + j3.28) = 79.5 + j164$

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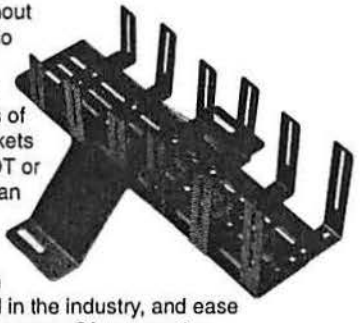

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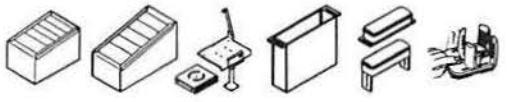

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